## Crystal-Structure Tools

Crystallography Online: Workshop on the use and applications of the structural and magnetic tools of the Bilbao Crystallographic Server


TRANSTRU


EQUIVSTRU


## POSSIBLE SYMMETRIES OF DISTORTED STRUCTURES

billbao crystallographic server

## Group-Subgroup Relations of Space Groups

```
SUBGROUPGRAPH Lattice of Maximal Subgroups
HERMANN Distribution of subgroups in conjugated classes
COSETS
WYCKSPLIT
MINSUP
SUPERGROUPS
CELLSUB
CELLSUPER
NONCHAR
COMMONSUBS
COMMONSUPER
INDEX
SUBGROUPS
Coset decomposition for a group-subgroup pair
The splitting of the Wyckoff Positions
Minimal Supergroups of Space Groups
Supergroups of Space Groups
List of subgroups for a given k-index.
List of supergroups for a given k-index.
Non Characteristic orbits.
Common Subgroups of Space Groups
Common Supergroups of Two Space Groups
Index of a group subgroup pair
Subgroups of a space group consistent with some given supercell, propagation vector(s) or irreducible representation(s)
```

Determine and explore online all possible symmetries that can result from the distortion of a parent structure of higher symmetry

## SUBGROUPS: https://www.cryst.ehu.es/cgi-bin/cryst/programs/subgrmag1 cell.pl

Subgroups: Subgroups compatible with a given supercell or some propagation vector(s).


Further limitations considering physical properties of the point groups

- Only centrosymmetric/non centrosymmetric groups
- Only polar/non polar groups
- Only proper ferroelastic phase transitions

- List of subgroups

O Graph of subgroups

Other alternatives to filtered the results of the program

## Example 1: Fullerene-cubane crystal

High-temperature phase


Nature Mat. 4, 764 (2005)


Low-temperature phase

Disordered fullerenes molecules $4 a(0,0,0)$
Disordered cubane molecules $4 b(1 / 2,1 / 2,1 / 2)$


Orthorhombic structure

$$
a_{o} \approx b_{o} \approx \frac{a_{c}}{\sqrt{2}} ; c_{o} \approx 2 a_{c}
$$

J. Phys. Chem. B 113, 2042 (2009)

Restrict the symmetry of the low-symmetry phase to a minimal set of most probable space groups

## Example 1: Fullerene-cubane crystal (a)

## Subgroups: Subgroups compatible with a given supercell or some propagation vector(s).

## Subgroups

The program Subgroups provides the possible subgroups of a space group which are possible for a given supercell. The program provides a list of the set of space groups or a graph showing the groupsubgroup hierarchy, grouped into conjugacy classes. More optional information about the classes or subgroups is also given.

Other alternatives for the input of the program:

- Instead of the whole set of subgroups, the output can be limited to subgroups having a chosen common subgroup of lowest symmetry, common point group of lowest symmetry, or groups which belong to a specific crystal class.
- Instead of a supercell, a set of modulation wave vectors can be given, including complete or partial


Enter the serial number of the space group:
choose it
225


[^0]O Graph of subgroups

## Example 1: Fullerene-cubane crystal (a)

List of subgroups that fulfill the given conditions
Get the subgroup-graph


99 subgroups

Input data
Subgroups of the space group
$F m \overline{3} m$ (N. 225)
P1 (N. 1)
(1/2,-1/2,0),(1/2,1/2,0),(0,0,2) No
Supercell given by: Centred supercell:

| 90 | Pm (No. 6) |  | $1 / 2$ $1 / 2$ 0 | $\begin{array}{r}0 \\ 0 \\ -2 \\ \hline\end{array}$ | $-1 / 2$ $1 / 2$ 0 | (re $\left.\begin{array}{r}0 \\ 0 \\ 1 / 2\end{array}\right) \mid$ | $96=4 \times 24$ | Conjugacy Class | Get irreps |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 91 | C2 (No. 5) |  | $\left(\begin{array}{l}1 \\ 0 \\ 0\end{array}\right.$ | 0 1 0 | 0 0 2 | $\left.\begin{array}{r}1 / 4 \\ 0 \\ 1 / 4\end{array}\right)$ | $96=4 \times 24$ | Conjugacy Class | Get irreps |
| 92 | C2 (No. 5) |  | $\left(\begin{array}{l}1 \\ 0 \\ 0\end{array}\right.$ | 0 1 0 | 0 0 2 | $\left.\begin{array}{r}0 \\ 0 \\ 1 / 2\end{array}\right)$ | $96=4 \times 24$ | Conjugacy Class | Get irreps |
| 93 | $P 2{ }_{1}$ (No. 4) |  | $\begin{array}{rr}1 / 2 & 1 / 2 \\ -1 / 2 & 1 \\ 0\end{array}$ | $1 / 2$ $1 / 2$ 0 | 0 0 2 | $\left.\begin{array}{r\|r\|}-1 / 8 \\ 1 / 8 \\ 1 / 4\end{array}\right)$ | $96=4 \times 24$ | Conjugacy Class | Get irreps |
| 94 | $P 2_{1}($ No. 4) |  | $1 / 2$ $1 / 2$ 0 | 0 0 -2 | $-1 / 2$ $1 / 2$ 0 | $\left.\begin{array}{l}0 \\ 0 \\ 0\end{array}\right)$ | $96=4 \times 24$ | Conjugacy Class | Get irreps |
| 95 | P2 (No. 3) |  | $\begin{array}{rr}1 / 2 & 1 / 2 \\ -1 / 2 & 1 \\ 0 & \end{array}$ | $1 / 2$ $1 / 2$ 0 | 0 0 2 | ( $\left.\begin{array}{r}0 \\ 0 \\ 1 / 2\end{array}\right)$ | $96=4 \times 24$ | Conjugacy Class | Get irreps |
| 96 | P2 (No. 3) |  | $1 / 2$ $1 / 2$ 0 | 0 0 -2 | $-1 / 2$ $1 / 2$ 0 | $\left.\begin{array}{l}0 \\ 0 \\ 0\end{array}\right)$ | $96=4 \times 24$ | Conjugacy Class | Get irreps |
| 97 | $P \overline{1}$ (No. 2) |  | $\begin{array}{rrr}1 / 2 & -1 / 2 \\ 1 / 2 & 1 \\ 0\end{array}$ | $1 / 2$ $1 / 2$ 0 | 0 0 2 | $\left.\begin{array}{r}0 \\ 1 / 4 \\ -1 / 4\end{array}\right)$ | $96=4 \times 24$ | Conjugacy Class | Get irreps |
| 98 | $P \overline{1}($ No. 2) |  | $\begin{array}{rrr}1 / 2 & -1 / 2 \\ 1 / 2 & 1 \\ 0\end{array}$ | $1 / 2$ $1 / 2$ 0 | 0 0 2 | ( $\left.\begin{array}{r}0 \\ 0 \\ 1 / 2\end{array}\right)$ | $96=4 \times 24$ | Conjugacy Class | Get irreps |
| 99 | P1 (No. 1) |  | $\begin{array}{rrr}1 / 2 & -1 / 2 \\ 1 / 2 & 1 \\ 0\end{array}$ | $1 / 2$ $1 / 2$ 0 | 0 0 2 | 0 0 | $192=4 \times 48$ | Conjugacy Class | Get irreps |

## Example 1: Fullerene-cubane crystal (a)




| 60 | $P 2_{1} 212$ (No. 18) | $\left(\begin{array}{r}1 / 2 \\ -1 / 2 \\ 0\end{array}\right.$ |  | 0 0 2 | ( $\left.\begin{array}{r}0 \\ 0 \\ -1 / 4\end{array}\right)$ | $48=4 \times 12$ | Conjugacy Class | Get irreps |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 61 | P222 ${ }_{1}$ (No. 17) | $\left(\begin{array}{l}1 / 2 \\ -1 / 2\end{array}\right.$ |  |  | re $\left.\begin{array}{r}0 \\ 1 / 2\end{array}\right)$ | $48=4 \times 12$ | Conjugacy Class | Get irreps |
| 62 | P222 (No. 16) | $\left(\begin{array}{r}1 / 2 \\ -1 / 2 \\ 0\end{array}\right.$ |  |  | $\left.\begin{array}{r}0 \\ 0 \\ 1 / 2\end{array}\right)$ | $48=4 \times 12$ | Coniugacy Class | Get irreps |

## Possible limitations of the subgroup list.

(Check only one option on the left and the specific value on the right)
(Check only one option on the left and the specific value on the right)

| O Lowest space group to consider | choose it |
| :--- | :--- |
| Lowest point group to consider | $------\quad$-- |
| Lowest crystal system to consider |  |
| Only maximal subgroups |  |

The list of maximal subgroups is reduced from 99 to 62

Most of them can be discarded symmetry higher than orthorhombic

Orthorhombic point groups: $222, \mathrm{~mm}$ or mmm

## Example 1: Fullerene-cubane crystal (b)

List of subgroups that fulfill the given conditions


| N | Group Symbol | Transformation matrix |  |  | Group-Subgroup index | Other members of the Conjugacy Class | irreps |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | $\mathrm{PH}_{2} / \mathrm{ncm}$ (No. 138) | $\left(\begin{array}{rr}1 / 2 \\ -1 / 2 & 1 / 2 \\ 0 & 1 / 2\end{array}\right.$ | $\begin{array}{rl}1 / 2 & 0 \\ 1 / 2 & 0 \\ 0 & 2\end{array}$ | ( $\left.\begin{array}{r}1 / 4 \\ 0 \\ -1 / 4\end{array}\right)$ | $12=4 \times 3$ | Conjugacy Class | Get irreps |
| 2 | $\mathrm{PA}_{2} / \mathrm{nmc}$ (No. 137) | $\left(\begin{array}{rl}1 / 2 \\ -1 / 2 & 1 / 2 \\ 0 & 1 / 2\end{array}\right.$ | $\begin{array}{rl}1 / 2 & 0 \\ 1 / 2 & 0 \\ 0 & 2\end{array}$ | $\left.\begin{array}{r}1 / 4 \\ \hline-1 / 4\end{array}\right)$ | $12=4 \times 3$ | Conjugacy Class | Get irreps |
| 3 | $\mathrm{P}_{2} / \mathrm{mcm}$ ( $\mathrm{No}$..132 ) | $\left(\begin{array}{rr}1 / 2 \\ -1 / 2 & 1 / 2 \\ 0 & 1 / 2\end{array}\right.$ | $\begin{array}{rr}1 / 2 & 0 \\ 1 / 2 & 0 \\ 0 & 2\end{array}$ | ( $\left.\begin{array}{r}0 \\ 0 \\ 1 / 2\end{array}\right)$ | $12=4 \times 3$ | Conjugacy Class | Get irreps |
| 4 | P42/mmc ( (No. 131) | $\left(\begin{array}{rr}1 / 2 \\ -1 / 2 & 1 / 2 \\ 0 & 1\end{array}\right.$ | $\begin{array}{rl}1 / 2 & 0 \\ 1 / 2 & 0 \\ 0 & 2\end{array}$ | ( $\left.\begin{array}{r}0 \\ 0 \\ 1 / 2\end{array}\right)$ | $12=4 \times 3$ | Conjugacy Class | Get irreps |
| 5 | P4/ncc (No. 130) | $\left(\begin{array}{rr}1 / 2 \\ -1 / 2 & 1 / 2 \\ 0 & 1\end{array}\right.$ | $\begin{array}{rl}1 / 2 & 0 \\ 1 / 2 & 0 \\ 0 & 2\end{array}$ | $\left.\begin{array}{r}1 / 4 \\ 0 \\ -1 / 4\end{array}\right)$ | $12=4 \times 3$ | Conjugacy Class | Get irreps |
| 6 | P4/nmm (No. 129) | $\left(\begin{array}{rr}1 / 2 \\ -1 / 2 & 1 \\ 0 & 1\end{array}\right.$ | $\begin{array}{rl}1 / 2 & 0 \\ 1 / 2 & 0 \\ 0 & 2\end{array}$ | $\left.\begin{array}{r}1 / 4 \\ 0 \\ -1 / 4\end{array}\right)$ | $12=4 \times 3$ | Conjugacy Class | Get irreps |
| 7 | P4/moc (No. 124) | $\left(\begin{array}{rr}1 / 2 \\ -1 / 2 & 1 / 2 \\ 0 & 1\end{array}\right.$ | $\begin{array}{rl}1 / 2 & 0 \\ 1 / 2 & 0 \\ 0 & 2\end{array}$ | ( $\left.\begin{array}{r}0 \\ 0 \\ 1 / 2\end{array}\right)$ | $12=4 \times 3$ | Conjugacy Class | Get irreps |
| 8 | P4/mmm (No. 123) | $\left(\begin{array}{rrr}1 / 2 \\ -1 / 2 & 1 / 2 \\ 0 & 1\end{array}\right.$ | $\begin{array}{rl}1 / 2 & 0 \\ 1 / 2 & 0 \\ 0 & 2\end{array}$ | ( $\left.\begin{array}{r}0 \\ 0 \\ 1 / 2\end{array}\right)$ | $12=4 \times 3$ | Conjugacy Class | Get irreps |
| 9 | Ccce (No.68) | $\left(\begin{array}{l}1 \\ 0 \\ 0\end{array}\right.$ | $\begin{array}{ll}0 & 0 \\ 1 & 0 \\ 0 & 2\end{array}$ | $\left.\begin{array}{r}1 / 4 \\ 0 \\ -1 / 4\end{array}\right)$ | $24=4 \times 6$ | Conjugacy Class | Get irreps |
| 10 | Cmme (No.67) | $\left(\begin{array}{l}1 \\ 0 \\ 0\end{array}\right.$ | $\begin{array}{ll}0 & 0 \\ 1 & 0 \\ 0 & 2\end{array}$ | ( $\left.\begin{array}{r}0 \\ -1 / 4\end{array}\right)$ | $24=4 \times 6$ | Conjugacy Class | Get irreps |
| 11 | Ccom (No.66) | $\left(\begin{array}{l}1 \\ 0 \\ 0\end{array}\right.$ | $\begin{array}{ll}0 & 0 \\ 1 & 0 \\ 0 & 2\end{array}$ | $\left.\begin{array}{r}0 \\ 0 \\ 1 / 2\end{array}\right)$ | 24=4x6 | Conjugacy Class | Get irreps |
| 12 | Cmmm (No.65) | $\left(\begin{array}{l}1 \\ 0 \\ 0\end{array}\right.$ | $\begin{array}{ll}0 & 0 \\ 1 & 0 \\ 0 & 2\end{array}$ | $\left.\begin{array}{r}0 \\ 0 \\ 1 / 2\end{array}\right)$ | $24=4 \times 6$ | Conjugacy Class | Get irreps |
| 13 | Cmoe (No.64) | $\left(\begin{array}{r}0 \\ -1 \\ 0\end{array}\right.$ | $\begin{array}{ll}1 & 0 \\ 0 & 0 \\ 0 & 2\end{array}$ | $\left.\begin{array}{r}1 / 4 \\ 0 \\ -1 / 4\end{array}\right)$ | 24=4x6 | Conjugacy Class | Get irrops |
| 14 | Cmam (No.63) | $\left(\begin{array}{r}0 \\ -1 \\ 0\end{array}\right.$ | 1 0 <br> 0 0 <br> 0 2 | $\left.\begin{array}{r}0 \\ 0 \\ 1 / 2\end{array}\right)$ | $24=4 \times 6$ | Conjugacy Class | Get irreps |
| 15 | Pnma (No.62) | $\left(\begin{array}{rr}0 & 1 / 2 \\ 0 & 1 / 2 \\ -2 & \end{array}\right.$ | (rr $\begin{array}{rr}1 / 2 & 1 / 2 \\ 1 / 2 & -1 / 2 \\ 0 & 0\end{array}$ | ( $\left.\begin{array}{r}1 / 4 \\ -1 / 4\end{array}\right)$ | 24=4x6 | Conjugacy Class | Get irrops |
| 16 | Pmmn (No. 59) | $\left(\begin{array}{cc}1 / 2 & -1 / 2 \\ 1 / 2 & 1 / 2 \\ 0 & \end{array}\right.$ | $\begin{array}{rl}-1 / 2 & 0 \\ 1 / 2 & 0 \\ 0 & 2\end{array}$ | $\left.\begin{array}{r}1 / 4 \\ -1 / 4\end{array}\right)$ | 24=4x6 | Conjugacy Class | Get irreps |
| 17 | Pcon (No. 56) | $\left(\begin{array}{cc}1 / 2 & -1 / 2 \\ 1 / 2 & 1 / 2 \\ 0 & \end{array}\right.$ | $\begin{array}{rr}-1 / 2 & 0 \\ 1 / 2 & 0 \\ 0 & 2\end{array}$ | $\left.\begin{array}{\|c}1 / 4 \\ -1 / 4\end{array}\right)$ | $24=4 \times 6$ | Conjugacy Class | Get irreps |
| 18 | Pmma (No. 51) | $\left(\begin{array}{rr}0 & 1 \\ 0 & 1 \\ -2 & \end{array}\right.$ | crer$1 / 2$ <br> $1 / 2$ | ( $\left.\begin{array}{r}0 \\ 0 \\ 1 / 2\end{array}\right)$ | $24=4 \times 6$ | Conjugacy Class | Get irrops |
| 19 | Pcom (No. 49) | $\left(\begin{array}{cc}1 / 2 & -1 / 2 \\ 1 / 2 & 1 / 2 \\ 0\end{array}\right.$ | $\begin{array}{rl}-1 / 2 & 0 \\ 1 / 2 & 0 \\ 0 & 2\end{array}$ | ( $\left.\begin{array}{r}0 \\ 0 \\ 1 / 2\end{array}\right)$ | $24=4 \times 6$ | Conjugacy Class | Get irreps |
| 20 | Pmmm (No. 47) | $\left(\begin{array}{rr}1 / 2 & 1 / 2 \\ -1 / 2 & 1 \\ 0 & 1\end{array}\right.$ | $\begin{array}{rr}1 / 2 & 0 \\ 1 / 2 & 0 \\ 0 & 2\end{array}$ | $\left.\begin{array}{r}0 \\ 0 \\ 1 / 2\end{array}\right)$ | 24=4x6 | Conjugacy Class | Get irreps |

## Example 1: Fullerene-cubane crystal (b)

Input data

Subgroups of the space group : Lowest point group to consider: Supercell given by: Centred supercell:

$F m \overline{3} m$ (N. 225)<br>mmm (N. 8)<br>1/2,-1/2,0),(1/2,1/2,0),(0,0,2)

Graph of subgroups that fulfill the given conditions


Get the subgraph between the group (or conjugacy class) with label $\qquad$ and the group (or conjugacy class) with label $\qquad$ according to these rules Get graph

Graph showing the group-subgroup hierarchy of these 20 subgroups

## Example 1: Fullerene-cubane crystal (b)

| N | Group Symbol | Transformation matrix |  |  | $\begin{gathered} \text { Group-Subgroup } \\ \text { index } \end{gathered}$ | Other members of the Conjugacy Class | irreps |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | P42/ncm (No. 138) | $\left(\begin{array}{rr}1 / 2 & 1 / 2 \\ -1 / 2 & 1 \\ 0\end{array}\right.$ | $\begin{array}{rr}1 / 2 & 0 \\ 1 / 2 & 0 \\ 0 & 2\end{array}$ | 1/4 0 | $12=4 \times 3$ | Conjugacy Class | Get irreps |
| 2 | P4 ${ }_{2} / \mathrm{nmc}$ (No. 137) | $\left(\begin{array}{rrr}1 / 2 & 1 / 2 \\ -1 / 2 & 1 \\ 0 & \end{array}\right.$ | $\begin{array}{rr}1 / 2 & 0 \\ 1 / 2 & 0 \\ 0 & 2\end{array}$ | 1/4 ${ }^{1 / 4}$ (1/4 | $12=4 \times 3$ | Conjugacy Class | Get irreps |
| 3 | $\mathrm{P} 4_{2} / \mathrm{mcm}$ ( No .132 ) | $\left(\begin{array}{rrr}1 / 2 & 1 / 2 \\ -1 / 2 & 1 \\ 0\end{array}\right.$ | $\begin{array}{rr}1 / 2 & 0 \\ 1 / 2 & 0 \\ 0 & 2\end{array}$ | ( $\left.\begin{array}{r}0 \\ 0 \\ 1 / 2\end{array}\right)$ | $12=4 \times 3$ | Conjugacy Class | Get irreps |
| 4 | P4 ${ }_{2} / \mathrm{mmc}$ ( No .131 ) | $\left(\begin{array}{rr}1 / 2 & 1 / 2 \\ -1 / 2 & 1 \\ 0\end{array}\right.$ | $1 / 2$ $1 / 2$ 0 | ( $\left.\begin{array}{r}0 \\ 0 \\ 1 / 2\end{array}\right)$ | $12=4 \times 3$ | Conjugacy Class | Get irreps |
| 5 | P4/ncc (No. 130) | $\left(\begin{array}{rrr}1 / 2 \\ -1 / 2 & 1 / 2 \\ 0 & 1\end{array}\right.$ | $\begin{array}{rr}1 / 2 & 0 \\ 1 / 2 & 0 \\ 0 & 2\end{array}$ | 1/4 0 | $12=4 \times 3$ | Conjugacy Class | Get irreps |
| 6 | P4/nmm (No. 129) | $\left(\begin{array}{rr}1 / 2 & 1 / 2 \\ -1 / 2 & 1 \\ 0\end{array}\right.$ | $\begin{array}{rr}1 / 2 & 0 \\ 1 / 2 & 0 \\ 0 & 2\end{array}$ | $\left.\begin{array}{r}1 / 4 \\ 0 \\ -1 / 4\end{array}\right)$ | $12=4 \times 3$ | Conjugacy Class | Get irreps |
| 7 | P4/mcc (No. 124) | $\left(\begin{array}{rll}1 / 2 & 1 / 2 \\ -1 / 2 & 1 \\ 0\end{array}\right.$ | $1 / 2$ $1 / 2$ 0 | $\left.\begin{array}{r}0 \\ 0 \\ 1 / 2\end{array}\right)$ | $12=4 \times 3$ | Conjugacy Class | Get irreps |
| 8 | P4/mmm (No. 123) | $\left(\begin{array}{rll}1 / 2 & 1 / 2 \\ -1 / 2 & 1 \\ 0\end{array}\right.$ | $1 / 2$ 0 <br> $1 / 2$ 0 <br> 0 2 | $\left.\begin{array}{r} 0 \\ 0 \\ 1 / 2 \end{array}\right)$ | $12=4 \times 3$ | Conjugacy Class | Get irreps |
| 9 | Ccce (No.68) | 1 0 0 | $\begin{array}{ll}0 & 0 \\ 1 & 0 \\ 0 & 2\end{array}$ | $\left.\begin{array}{r}1 / 4 \\ 1 / 4 \\ -1 / 4\end{array}\right)$ | $24=4 \times 6$ | Conjugacy Class | Get irreps |
| 10 | Cmme (No.67) | $\left(\begin{array}{l}1 \\ 0 \\ 0\end{array}\right.$ | $\begin{array}{ll}0 & 0 \\ 1 & 0 \\ 0 & 2\end{array}$ | ( $\left.\begin{array}{c}1 / 4 \\ -1 / 4\end{array}\right)$ | $24=4 \times 6$ | Conjugacy Class | Get irreps |
| 11 | Ccom (No. 66) | $\left(\begin{array}{l}1 \\ 0 \\ 0\end{array}\right.$ | 0 0 <br> 1 0 <br> 0 2 | $\left.\begin{array}{r}0 \\ 0 \\ 1 / 2\end{array}\right)$ | $24=4 \times 6$ | Conjugacy Class | Get irreps |
| 12 | Cmmm (No.65) | $\left(\begin{array}{l}1 \\ 0 \\ 0\end{array}\right.$ | $\begin{array}{ll}0 & 0 \\ 1 & 0 \\ 0 & 2\end{array}$ | ( $\left.\begin{array}{r}0 \\ 0 \\ 1 / 2\end{array}\right)$ | $24=4 \times 6$ | Conjugacy Class | Get irreps |
| 13 | Cmce (No. 64) | ( $\begin{array}{r}0 \\ -1 \\ 0\end{array}$ | $\begin{array}{ll}1 & 0 \\ 0 & 0 \\ 0 & 2\end{array}$ |  | $24=4 \times 6$ | Conjugacy Class | Get irreps |
| 14 | Cmcm (No.63) | $\left(\begin{array}{r}0 \\ -1 \\ 0\end{array}\right.$ | $\begin{array}{ll}1 & 0 \\ 0 & 0 \\ 0 & 2\end{array}$ | $\left.\begin{array}{r}0 \\ 0 \\ 1 / 2\end{array}\right)$ | $24=4 \times 6$ | Conjugacy Class | Get irreps |
| 15 | Pnma (No. 62) | ( $\begin{array}{rrr}0 & 1 / 2 \\ 0 & 1 \\ -2 & 1\end{array}$ | $\begin{array}{rrr}1 / 2 & 1 / 2 \\ 1 / 2 & -1 / 2 \\ 0 & 0\end{array}$ | $\left.\begin{array}{r}0 \\ 1 / 4 \\ -1 / 4\end{array}\right)$ | $24=4 \times 6$ | Conjugacy Class | Get irreps |
| 16 | Pmmn (No. 59) | $\left(\begin{array}{cc}1 / 2 & -1 / 2 \\ 1 / 2 & 1 / 2 \\ 0 & \end{array}\right.$ | $\begin{array}{rr}1 / 2 & 0 \\ 1 / 2 & 0 \\ 0 & 2\end{array}$ | $\left.\begin{array}{r}1 / 4 \\ -1 / 4\end{array}\right)$ | $24=4 \times 6$ | Conjugacy Class | Get irreps |
| 17 | Pcon (No. 56) | $\left(\begin{array}{rrr}1 / 2 & -1 / 2 \\ 1 / 2 & 1 \\ 0\end{array}\right.$ | $\begin{array}{rr}1 / 2 & 0 \\ 1 / 2 & 0 \\ 0 & 2\end{array}$ | $\left.\begin{array}{r}0 \\ 1 / 4 \\ -1 / 4\end{array}\right)$ | $24=4 \times 6$ | Conjugacy Class | Get irreps |
| 18 | Pmma (No. 51) | $\left(\begin{array}{rrr}0 & 1 / 2 \\ 0 & 1 \\ -2 & \end{array}\right.$ | $\begin{array}{rrr}1 / 2 & 1 / 2 \\ 1 / 2 & -1 / 2 \\ 0 & 0\end{array}$ | ren $\left.\begin{array}{r}0 \\ 1 / 2\end{array}\right)$ | $24=4 \times 6$ | Conjugacy Class | Get irreps |
| 19 | Pcom (No. 49) | $\left(\begin{array}{c}1 / 2 \\ 1 / 2 \\ \text { 2 }\end{array} 1\right.$ | $\begin{array}{rr}1 / 2 & 0 \\ 1 / 2 & 0 \\ 0 & 2\end{array}$ | re $\left.\begin{array}{r}0 \\ 1 / 2\end{array}\right)$ | $24=4 \times 6$ | Conjugacy Class | Get irreps |
| 20 | Pmmm (No.47) | $\left(\begin{array}{rll}1 / 2 & 1 / 2 \\ -1 / 2 & 1 \\ 0\end{array}\right.$ | $\begin{array}{rr}1 / 2 & 0 \\ 1 / 2 & 0 \\ 0 & 2\end{array}$ | ( $\left.\begin{array}{r}0 \\ 0 \\ 1 / 2\end{array}\right)$ | $24=4 \times 6$ | Conjugacy Class | Get irreps |

Tetragonal subgroups

C centered orthorhombic

The unit cell of the LS-phase is known to primitive orthorhombic

6 possible symmetries

## Example 1: Fullerene-cubane crystal (c)

| 15 | Pnma (No.62) |  |  | $1 /$ | 1/2 | $1 / 2$ $-1 / 2$ 0 | ( $\left.\begin{array}{r}0 \\ 1 / 4 \\ -1 / 4\end{array}\right)$ | $24=4 \times 6$ | Conjugacy Class | Get irreps |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 16 | Pmmn (No. 59) |  | 1/2 | ${ }_{1}^{1 /}$ | $1 / 2$ $1 / 2$ 0 | 0 0 2 | ( $\left.\begin{array}{r}0 \\ -1 / 4\end{array}\right)$ | $24=4 \times 6$ | Coniugacy Class | Get irreps |
| 17 | Pcon (No. 56) |  | $1 / 2$ | $1 /$ | $1 / 2$ $1 / 2$ 0 | 2 | $\left.\begin{array}{r}0 \\ 1 / 4 \\ -1 / 4\end{array}\right)$ | $24=4 \times 6$ | Conjugacy Class | Get irreps |
| 18 | Pmma (No. 51) |  | -2 | $1 /$ | $1 / 2$ $1 / 2$ 0 | $1 / 2$ $-1 / 2$ 0 | r $\left.\begin{array}{r}0 \\ 1 / 2\end{array}\right)$ | $24=4 \times 6$ | Conjugacy Class | Get irreps |
| 19 | Pcom (No. 49) |  | $1 / 2$ $1 / 2$ | $1 /$ | -1/2 | 0 <br> 0 <br> 2 | $\left.\begin{array}{r}0 \\ 0 \\ 1 / 2\end{array}\right)$ | $24=4 \times 6$ | Conjugacy Class | Get irreps |
| 20 | Pmmm (No. 47) |  | 1/2 | $1 /$ | $1 / 2$ $1 / 2$ 0 | 0 <br> 2 | $\left.\begin{array}{r}0 \\ 0 \\ 1 / 2\end{array}\right)$ | 24=4x6 | Conjugacy Class | Get irreps |

The irreducible representations of the parent structure that are compatible with this specific symmetry for the distorted structure

List of physically irreducible representations and order parameters between a parent group and a given subgroup.

Input data

| Group $\rightarrow$ subgroup | Transformation matrix |
| :---: | :---: |
| $F m \overline{3} m(\mathbf{N} .225) \rightarrow \operatorname{Pnma}(\mathrm{N} .62)$ | $\left(\begin{array}{rrrr}0 \\ 0 & 1 / 2 & 1 / 2 & 0 \\ 0 & 1 / 2 & -1 / 2 & 1 / 4 \\ -2 & 0 & 0 & -1 / 4\end{array}\right)$ |

Representations and order parameters

The symmetry break
$F m \overline{3} m \rightarrow$ Pnma can be realized through a Landau type phase transition

Show the graph of isotropy subgroups

| k-vectors | irreps and order parameters | isotropy subgroup transformation matrix | link to the irreps |
| :---: | :---: | :---: | :---: |
| GM: $(0,0,0)$ | $\mathrm{GM}_{1}^{+}$: (a) | $\begin{gathered} \hline F m \overline{3} m(\text { No. 225) } \\ a, b, c ; 0,0,0 \end{gathered}$ |  |
|  | $\mathrm{GM}_{3}{ }^{\text {a }}$ (a,0) | $\begin{gathered} 14 / \mathrm{mmm} \text { (No. 139) } \\ \mathrm{a} / 2-\mathrm{b} / 2, \mathrm{a} / 2+\mathrm{b} / 2, \mathrm{c} ; 0,0,0 \end{gathered}$ | matrices of the irreps |
|  | $\mathrm{GM}_{5}{ }^{+}:(\mathrm{a}, 0,0)$ | $\begin{gathered} \operatorname{Immm}(\text { No. 71) } \\ \mathrm{a} / 2+\mathrm{b} / 2,-1 / 2 \mathrm{a}+\mathrm{b} / 2, \mathrm{c} ; 0,0,0 \end{gathered}$ |  |
| DT: $(0,1 / 2,0)(1 / 2,0,0)(0,0,1 / 2)$ | $D T_{5}:(0,0,0,0,0,0,0,0, a, 0,0, a)$ | $\begin{gathered} \text { Pnma (No. 62) } \\ -2 \mathrm{c}, \mathrm{a} / 2+\mathrm{b} / 2, \mathrm{a} / 2-\mathrm{b} / 2 ; 0,1 / 4,-1 / 4 \end{gathered}$ | matrices of the irreps |
| X: $(0,1,0)(1,0,0)(0,0,1)$ | $\mathrm{X}_{2}{ }^{-}$( $0,0, \mathrm{a}$ ) | $\begin{gathered} P 4_{2} / n m c(\text { No. 137 }) \\ a / 2-b / 2, a / 2+b / 2, c ; 0,1 / 4,1 / 4 \end{gathered}$ | matrices of the irreps |
|  | $X_{3}{ }^{-}:(0,0, a)$ | $\begin{gathered} \text { P4/nmm (No. 129) } \\ \mathrm{a} / 2-\mathrm{b} / 2, \mathrm{a} / 2+\mathrm{b} / 2, \mathrm{c} ; 1 / 4,0,1 / 4 \end{gathered}$ | marices ofthe ireps |

## Example 1: Fullerene-cubane crystal (d)

List of physically irreducible representations and order parameters between a parent group and a given subgroup.

Input data

| Group $\rightarrow$ subgroup | Transformation matrix |
| :---: | :---: | :---: |
| Fm $\overline{3} m($ N. 225 $) \rightarrow$ Pnma (N. 62) | $\left(\begin{array}{rrrr} \\ & \left(\begin{array}{rrrr}1 / 2 & 1 / 2 & 0 \\ 0 & 1 / 2 & -1 / 2 & 1 / 4 \\ -2 & 0 & 0 & -1 / 4\end{array}\right) \\ \hline \hline\end{array}\right.$ |

Representations and order parameters

Show the graph of isotropy subgroups

| k-vectors | irreps and order parameters | isotropy subgroup transformation matrix | link to the irreps |
| :---: | :---: | :---: | :---: |
| GM: $(0,0,0)$ | $\mathrm{GM}_{1}{ }^{+}$: ${ }^{\text {a }}$ | $\begin{gathered} \hline \text { Fm } \overline{3} m \text { (No. 225) } \\ \text { a,b,c;0,0,0 } \end{gathered}$ |  |
|  | $\mathrm{GM}_{3}{ }^{\text {a }}$ ( $\left.\mathrm{a}, 0\right)$ | $\begin{gathered} 14 / \mathrm{mmm} \text { (No. 139) } \\ \mathrm{a} / 2-\mathrm{b} / 2, \mathrm{a} / 2+\mathrm{b} / 2, \mathrm{c} ; 0,0,0 \end{gathered}$ | matrices of the irreps |
|  | $\mathrm{GM}_{5}^{+}:(\mathrm{a}, 0,0)$ | $\begin{gathered} \text { Immm (No. 71) } \\ \mathrm{a} / 2+\mathrm{b} / 2,-1 / 2 \mathrm{a}+\mathrm{b} / 2, \mathrm{c} ; 0,0,0 \end{gathered}$ |  |
| DT: $(0,1 / 2,0)(1 / 2,0,0)(\mathbf{0 , 0 , 1 / 2 )}$ | $D T_{5}:(0,0,0,0,0,0,0,0, a, 0,0, a)$ | $\begin{gathered} \text { Pnma (No. 62) } \\ -2 c, a / 2+b / 2, a / 2-b / 2 ; 0,1 / 4,-1 / 4 \end{gathered}$ | matrices of the irreps |
| X: $(0,1,0)(1,0,0)(0,0,1)$ | $\mathrm{X}_{2}=(0,0, a)$ | $\begin{gathered} P 4_{2} / n m c(\text { No. 137 }) \\ \mathrm{a} / 2-\mathrm{b} / 2, \mathrm{a} / 2+\mathrm{b} / 2, \mathrm{c} ; 0,1 / 4,1 / 4 \end{gathered}$ | atrices of the irreps |
|  | $\mathrm{X}_{3}{ }^{-}(0,0, a)$ | $\begin{gathered} \text { P4/nmm (No. 129) } \\ \text { a/2-b/2,a/2+b/2,c;1/4,0,1/4 } \end{gathered}$ | matrices of the irreps |



## Example 1: Fullerene-cubane crystal (e)

Use the option Get irreps for the other possible symmetries

| 15 | Pnma (No. 62) |  | ( | 0 0 -2 | $1 / 2$ $1 / 2$ 0 | $1 / 2$ $-1 / 2$ 0 | $\left.\begin{array}{r}0 \\ 1 / 4 \\ -1 / 4\end{array}\right)$ | $24=4 \times 6$ | Conjugacy Class | Get irreps |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 16 | Pmmn (No. 59) |  | $\left(\begin{array}{l}1 / 2 \\ 1 /\end{array}\right.$ | $1 / 2$ $1 / 2$ 0 | $-1 / 2$ $1 / 2$ 0 | 0 0 2 | $\left.\begin{array}{r}0 \\ 1 / 4 \\ -1 / 4\end{array}\right)$ | $24=4 \times 6$ | Conjugacy Class | Get irreps |
| 17 | Pcon (No. 56) |  | $\left(\begin{array}{l}1 / 2 \\ 1\end{array}\right.$ | $1 / 2$ $1 / 2$ 0 | $-1 / 2$ $1 / 2$ 0 | 0 0 2 | $\left.\begin{array}{r}0 \\ 1 / 4 \\ -1 / 4\end{array}\right)$ | $24=4 \times 6$ | Conjugacy Class | Get irreps |
| 18 | Pmma (No. 51) |  | - | 0 0 -2 | $1 / 2$ $1 / 2$ 0 | $1 / 2$ $-1 / 2$ 0 | $\left.\begin{array}{r}0 \\ 0 \\ 1 / 2\end{array}\right)$ | $24=4 \times 6$ | Conjugacy Class | Get irreps |
| 19 | Pccm (No. 49) |  | $\left(\begin{array}{l}1 / 2 \\ 1\end{array}\right.$ | $1 / 2$ $1 / 2$ 0 | $-1 / 2$ $1 / 2$ 0 | 0 0 2 | $\left.\begin{array}{r}0 \\ 0 \\ 1 / 2\end{array}\right)$ | $24=4 \times 6$ | Conjugacy Class | Get irreps |
| 20 | Pmmm (No. 47) |  | $\left(\begin{array}{c}1 / \\ -1\end{array}\right.$ | $1 / 2$ $1 / 2$ 0 | $1 / 2$ $1 / 2$ 0 | 0 0 2 | ( $\left.\begin{array}{r}0 \\ 0 \\ 1 / 2\end{array}\right)$ | $24=4 \times 6$ | Conjugacy Class | Get irreps |

Only two of the symmetries can be result of a single irrep order parameter (fulfill the Landau condition)

$$
\begin{aligned}
& F m \overline{3} m \xrightarrow{\text { DT5 }} \operatorname{Pnma}(-2 \boldsymbol{c},-1 / 2 \boldsymbol{a}+1 / 2 \boldsymbol{b}, 1 / 2 \boldsymbol{a}-1 / 2 \boldsymbol{b} ; 0,1 / 4,-1 / 4) \\
& \text { DT5 } \\
& F m \overline{3} m \rightarrow P m m a(-2 \boldsymbol{c}, 1 / 2 a+1 / 2 \boldsymbol{b}, 1 / 2 \boldsymbol{a}-1 / 2 \boldsymbol{b} ; 0,0,1 / 2)
\end{aligned}
$$

## Example 1: Fullerene-cubane crystal (f)

List of physically irreducible representations and order parameters between a parent group and a given subgroup.

Input data

| Group $\rightarrow$ subgroup | Transformation matrix |
| :---: | :---: |
| $F m \overline{3} m$ (N. 225) $\rightarrow$ Pmmn (N. 59) | $\left(\begin{array}{rrrr}1 / 2 & -1 / 2 & 0 & 0 \\ 1 / 2 & 1 / 2 & 0 & 1 / 4 \\ 0 & 0 & 2 & -1 / 4\end{array}\right)$ |

Representations and order parameters

Show the graph of isotropy subgroups

| k-vectors | irreps and order parameters | isotropy subgroup transformation matrix | link to the irreps |
| :---: | :---: | :---: | :---: |
| GM: $(0,0,0)$ | $\mathrm{GM}_{1}{ }^{+}$: (a) | $\begin{gathered} \hline F m \overline{3} m(\text { No. 225) } \\ \text { a,b,c;0,0,0 } \\ \hline \end{gathered}$ |  |
|  | $\mathrm{GM}_{3}{ }^{\text {a }}$ ( $\left.\mathrm{a}, 0\right)$ | $\begin{gathered} 14 / \mathrm{mmm} \text { (No. 139) } \\ \mathrm{a} / 2-\mathrm{b} / 2, \mathrm{a} / 2+\mathrm{b} / 2, \mathrm{c} ; 0,0,0 \end{gathered}$ | matrices of the irreps |
|  | $\mathrm{GM}_{5}^{+}$: $(\mathrm{a}, 0,0)$ | $\begin{gathered} \text { Immm (No. 71) } \\ \mathrm{a} / 2+\mathrm{b} / 2,-1 / 2 \mathrm{a}+\mathrm{b} / 2, \mathrm{c} ; 0,0,0 \end{gathered}$ |  |
| DT: $(0,1 / 2,0)(1 / 2,0,0)(\mathbf{0 , 0 , 1 / 2 )}$ | $D T_{1}:(0,0,0,0, a, a)$ | $\begin{array}{\|c\|} \hline P 4 / n m m \text { (No. 129) } \\ \mathrm{a} / 2-\mathrm{b} / 2, \mathrm{a} / 2+\mathrm{b} / 2,2 \mathrm{c} ; 1 / 4,0,-1 / 4 \end{array}$ |  |
|  | $\mathrm{DT}_{3}:(0,0,0,0, \mathrm{a},-\mathrm{a})$ | $\begin{array}{\|c\|} P 4_{2} / n m c(\text { No. 137) } \\ \mathrm{a} / 2-\mathrm{b} / 2, \mathrm{a} / 2+\mathrm{b} / 2,2 \mathrm{c} ; 1 / 4,0,-1 / 4 \end{array}$ | matrices of the irreps |
| $\mathrm{X}:(0,1,0)(1,0,0)(0,0,1)$ | $\mathrm{X}_{2}{ }^{-}:(0,0, a)$ | $\begin{gathered} P 4_{2} / n m c(\text { No. 137 ) } \\ \mathrm{a} / 2-\mathrm{b} / 2, \mathrm{a} / 2+\mathrm{b} / 2, \mathrm{c} ; 0,1 / 4,1 / 4 \end{gathered}$ | matrices of the irreps |
|  | $\mathrm{X}_{3}{ }^{-}(0,0, a)$ | $\begin{gathered} \text { P4/nmm (No. 129) } \\ \text { a/2-b/2,a/2+b/2,c;1/4,0,1/4 } \end{gathered}$ | matrices of the ireps |



## Example 1: Fullerene-cubane crystal (g)

Do the same process as in the previous step for the other 3 possible symmetries

| Group $\rightarrow$ subgroup | Transformation matrix |
| :---: | :---: | :---: |
| $F m \overline{3} m$ (N. 225) $\rightarrow \operatorname{Pccn}(\mathrm{N} .56)$ | $\left(\begin{array}{rrrr}1 / 2 & -1 / 2 & 0 & 0 \\ 1 / 2 & 1 / 2 & 0 & 1 / 4 \\ 0 & 0 & 2 & -1 / 4\end{array}\right)$ |



| Group $\rightarrow$ subgroup | Transformation matrix |  |  |
| :---: | :---: | :---: | :---: |
| $F m \overline{3} m$ ( N .225 ) $\rightarrow$ Pmmm (N. 47) | $\left(\begin{array}{rr}1 / 2 & 1 / 2 \\ -1 / 2 & 1 / 2 \\ 0 & 0\end{array}\right.$ | 0 0 2 | $\left.\begin{array}{r}0 \\ 0 \\ 1 / 2\end{array}\right)$ |



## Example 1: Fullerene-cubane crystal (h)


$\nabla$ Optional: Show only subgroups that can be the result of a Landau-type transition (single irrep order parameter).


This option allows to discard all possible symmetries that cannot be reached by the action of a single irrep distortion

## Example 1: Fullerene-cubane crystal (h)



This condition reduces the number of possible distinct symmetries from 99 to 31

From the six-non-centered subgroups with point group $m m m$, only the subroups of type Pnma and Pmma appear here

## Example 2: Parent space group Pnma

Let us suppose that we observe a structure with symmetry Pnma, which exhibits when lowering the temperature a phase transition. Diffraction experiments in the low symmetry phase give evidence of superstructure reflections, which can be indexes as ( $h, k, l+1 / 2$ ).

This additional diffraction peaks indicates a distortion $\Rightarrow \begin{gathered}\text { duplication of } \\ \text { the c parameter }\end{gathered}$

Modulation wave vector ( $0,0,1 / 2$ )

We wish to know the possible space group symmetries that this low temperature phase can have, in order to construct structural models that could fit the diffraction data

## Example 2: Parent space group Pnma (a)

Subgroups: Subgroups compatible with a given supercell or some propagation vector(s).

## Subgroups

The program Subgroups provides the possible subgroups of a space group which are possible for a given supercell. The program provides a list of the set of space groups or a graph showing the groupsubgroup hierarchy, grouped into conjugacy classes. More optional information about the classes or subgroups is also given.

Other alternatives for the input of the program:

- Instead of the whole set of subgroups, the output can be limited to subgroups having a chosen common subgroup of lowest symmetry, common point group of lowest symmetry, or groups which belong to a specific crystal class.

Enter the serial number of the space group:
choose it
62

Introduce the supercell


## O List of subgroups

O Graph of subgroups

## Example 2: Parent space group Pnma (a)

Input data

Subgroups of the space group : Lowest space group to consider Supercell given by:
Centred supercell:

Pnma (N. 62)
$P 1$ (N. 1) (1,0,0),(0,1,0),(0,0,2)

List of subgroups that fulfill the given conditions
Get the subgroup-graph

| N | Group Symbol | Transformation matrix |  |  |  |  | Group-Subgroup index | Other members of the Conjugacy Class | irreps |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | Pca2 ${ }_{1}$ (No. 29) |  | 0 0 -2 | 0 1 0 | 1 0 0 | $\left.\begin{array}{r}0 \\ 1 / 4 \\ -1 / 4\end{array}\right)$ | $4=2 \times 2$ | Conjugacy Class | Get irreps |
| 2 | Pmc2 ${ }_{1}$ (No. 26) |  | 0 -1 0 | 0 0 -2 | 1 0 0 | $\left.\begin{array}{r}0 \\ 1 / 4 \\ -1 / 4\end{array}\right)$ | $4=2 \times 2$ | Conjugacy Class | Get irreps |
| 3 | $P 2_{1} / \mathrm{C}$ (No. 14) |  | $\left(\begin{array}{l}1 \\ 0 \\ 0\end{array}\right.$ | 1 0 | 0 0 2 | ( $\left.\begin{array}{l}0 \\ 0 \\ 0\end{array}\right)$ | $4=2 \times 2$ | Conjugacy Class | Get irreps |
| 4 | $P 2_{1} / m$ (No. 11) |  | $\left(\begin{array}{l}1 \\ 0 \\ 0\end{array}\right.$ | 1 0 | 0 0 2 | ( $\left.\begin{array}{r}0 \\ 0 \\ 1 / 2\end{array}\right)$ | $4=2 \times 2$ | Conjugacy Class | Get irreps |
| 5 | Pc (No. 7) |  | ( $\begin{array}{r}0 \\ -1 \\ 0\end{array}$ | 0 0 -2 | 1 0 0 | ( $\left.\begin{array}{r}0 \\ 0 \\ -1 / 4\end{array}\right)$ | $8=2 \times 4$ | Conjugacy Class | Get irreps |
| 6 | Pc (No. 7) |  | $\left(\begin{array}{l}1 \\ 0 \\ 0\end{array}\right.$ |  | 0 0 2 | $\left.\begin{array}{r}0 \\ 1 / 4 \\ 0\end{array}\right)$ | $8=2 \times 4$ | Conjugacy Class | Get irreps |
| 7 | Pm (No. 6) |  | $\left(\begin{array}{l}1 \\ 0 \\ 0\end{array}\right.$ | 0 1 0 | 0 0 2 | $\left.\begin{array}{r}0 \\ 1 / 4 \\ 0\end{array}\right)$ | $8=2 \times 4$ | Conjugacy Class | Get irreps |
| 8 | $P 2_{1}$ (No. 4) |  | $\left(\begin{array}{l}1 \\ 0 \\ 0\end{array}\right.$ | 0 | 0 0 2 | $\left.\begin{array}{r}0 \\ 0 \\ 1 / 2\end{array}\right)$ | $8=2 \times 4$ | Conjugacy Class | Get irreps |
| 9 | $P 2_{1}$ (No. 4) |  | 0 -1 0 | 1 0 0 | 0 0 2 | $\left.\begin{array}{r}0 \\ 1 / 4 \\ -1 / 4\end{array}\right)$ | $8=2 \times 4$ | Conjugacy Class | Get irreps |
| 10 | $P \overline{1}$ (No. 2) |  | $\left(\begin{array}{l}1 \\ 0 \\ 0\end{array}\right.$ | 0 1 0 | 0 0 2 | ( $\left.\begin{array}{r}0 \\ 0 \\ 1 / 2\end{array}\right)$ | $8=2 \times 4$ | Conjugacy Class | Get irreps |
| 11 | P1 (No. 1) |  | $\left(\begin{array}{l}1 \\ 0 \\ 0\end{array}\right.$ | 0 1 0 | 0 0 2 | 0 | $16=2 \times 8$ | Conjugacy Class | Get irreps |

Without any additional restriction the program lists 11 possible space group symmetries

Example 2: Parent space group Pnma (b)


## Example 2: Parent space group Pnma (b)

Go back to the input page


List of subgroups that fulfill the given conditions

| Get the subgroup-graph |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| N | Group Symbol | Transformation matrix |  |  |  | Group-Subgroup index | Other members of the Conjugacy Class | irreps |
| 1 | Pca2 ${ }_{1}$ (No. 29) | $\left(\begin{array}{r}0 \\ 0 \\ -2\end{array}\right.$ | 0 1 0 | 1 0 | $\left.\begin{array}{r}0 \\ 1 / 4 \\ -1 / 4\end{array}\right)$ | $4=2 \times 2$ | Conjugacy Class | Get irreps |
| 2 | Pmc2 ${ }_{1}$ (No. 26) | $\left(\begin{array}{r}0 \\ -1 \\ 0\end{array}\right.$ | 0 0 -2 | 1 0 0 | $\left.\begin{array}{r}0 \\ 1 / 4 \\ -1 / 4\end{array}\right)$ | $4=2 \times 2$ | Conjugacy Class | Get irreps |
| 3 | $P 2{ }_{1} / \mathrm{c}$ (No. 14) | $\left(\begin{array}{l}1 \\ 0 \\ 0\end{array}\right.$ | 0 1 0 | 0 0 2 | $\left.\begin{array}{l}0 \\ 0 \\ 0\end{array}\right)$ | $4=2 \times 2$ | Conjugacy Class | Get irreps |
| 4 | $P 2{ }_{1} / m$ (No. 11) | $\left(\begin{array}{l}1 \\ 0 \\ 0\end{array}\right.$ | 0 1 0 | 0 0 2 | $\left.\begin{array}{r}0 \\ 0 \\ 1 / 2\end{array}\right)$ | $4=2 \times 2$ | Conjugacy Class | Get irreps |



## Example 2: Parent space group Pnma (c)

Go back to the input page

| Possible limitations of the subgroup list. |  |  |
| :---: | :---: | :---: |
| (Check only one option on the (Check only one option on the let | the right) |  |
| O Lowest space group to consider |  |  |
|  | 1 |  |
| O Lowest point group to consider | ------ | $\checkmark$ |
| - Lowest crystal system to consider | ------ | $\checkmark$ |
| $\bigcirc$ Only maximal subgroups |  |  |

Further limitations considering physical properties of the point groups

- Only centrosymmetric/non centrosymmetric groups
- Only polar/non polar groups
all
all $\checkmark$
- Only proper ferroelastic phase transitions


List of subgroups that fulfill the given conditions
Get the subgroup-graph

| N | Group Symbol | Transformation matrix |  |  |  |  | Group-Subgroup index | Other members of the Conjugacy Class | irreps |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | Pca2 ${ }_{1}$ (No. 29) |  | 0 0 -2 | 0 1 | 1 0 | ( $\left.\begin{array}{r}0 \\ 1 / 4 \\ -1 / 4\end{array}\right)$ | $4=2 \times 2$ | Conjugacy Class | Get irreps |
| 2 | $\mathrm{Pmc2}_{1}($ No. 26) |  | 0 -1 0 | 0 0 -2 | 1 | ( $\left.\begin{array}{r}0 \\ 1 / 4 \\ -1 / 4\end{array}\right)$ | $4=2 \times 2$ | Conjugacy Class | Get irreps |
| 3 | Pc (No. 7) |  | 0 -1 0 | 0 0 -2 | 0 | ( $\left.\begin{array}{r}0 \\ 0 \\ -1 / 4\end{array}\right)$ | $8=2 \times 4$ | Conjugacy Class | Get irreps |
| 4 | Pc (No. 7) |  | $\left(\begin{array}{l}1 \\ 0 \\ 0\end{array}\right.$ | 0 1 0 | 0 0 2 | ( $\left.\begin{array}{r}0 \\ 1 / 4 \\ 0\end{array}\right)$ | $8=2 \times 4$ | Conjugacy Class | Get irreps |
| 5 | Pm (No. 6) |  | $\left(\begin{array}{l}1 \\ 0 \\ 0\end{array}\right.$ | 0 | 0 0 2 | $\left.\begin{array}{r}0 \\ 1 / 4 \\ 0\end{array}\right)$ | $8=2 \times 4$ | Conjugacy Class | Get irreps |
| 6 | $P 2_{1}$ (No. 4) |  | $\left(\begin{array}{l}1 \\ 0 \\ 0\end{array}\right.$ | 0 1 0 | 0 0 2 | ( $\left.\begin{array}{r}0 \\ 0 \\ 1 / 2\end{array}\right)$ | $8=2 \times 4$ | Conjugacy Class | Get irreps |
| 7 | $P 2_{1}$ (No. 4) |  | 0 -1 0 | 1 0 0 | 0 0 2 | $\left.\begin{array}{r}0 \\ 1 / 4 \\ -1 / 4\end{array}\right)$ | $8=2 \times 4$ | Conjugacy Class | Get irreps |
| 8 | P1 (No. 1) |  | $\left(\begin{array}{l}1 \\ 0 \\ 0\end{array}\right.$ | 0 1 0 | 0 0 2 | $\left.\begin{array}{l}0 \\ 0 \\ 0\end{array}\right)$ | $16=2 \times 8$ | Conjugacy Class | Get irreps |

## Example 2: Parent space group Pnma (d)

Go back to the input page

## Possible limitations of the subgroup list.

(Check only one option on the left and the specific value on the right) (Check only one option on the left and the specific value on the right)

- Lowest space group to consider
- Lowest point group to consider
- Lowest crystal system to consider


O Only maximal subgroups

Further limitations considering physical properties of the point groups

- Only centrosymmetric/non
centrosymmetric groups
- Only polar/non polar groups
- Only proper ferroelastic phase transitions


| N | Group Symbol | Transformation matrix |  |  |  |  | Group-Subgroup index | Other members of the Conjugacy Class | irreps |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | Pca2 ${ }_{1}$ (No. 29) |  | 0 0 -2 | 0 1 | 1 0 0 | $\left.\begin{array}{r}0 \\ 1 / 4 \\ -1 / 4\end{array}\right)$ | $4=2 \times 2$ | Conjugacy Class | Get irreps |
| 2 | Pmc2 ${ }_{1}$ (No. 26) |  | 0 -1 0 | 0 0 -2 | 1 0 0 | $\left.\begin{array}{r}0 \\ 1 / 4 \\ -1 / 4\end{array}\right)$ | $4=2 \times 2$ | Conjugacy Class | Get irreps |
| 3 | $P 2_{1} / \mathrm{C}$ (No. 14) |  | $\left(\begin{array}{l}1 \\ 0 \\ 0\end{array}\right.$ | 0 1 0 | 0 0 2 | $\left.\begin{array}{l}0 \\ 0 \\ 0\end{array}\right)$ | $4=2 \times 2$ | Conjugacy Class | Get irreps |
| 4 | $P 2{ }_{1} / m$ (No. 11) |  | $\left(\begin{array}{l}1 \\ 0 \\ 0\end{array}\right.$ | 0 1 | 0 0 2 | ( $\left.\begin{array}{r}0 \\ 0 \\ 1 / 2\end{array}\right)$ | $4=2 \times 2$ | Conjugacy Class | Get irreps |
| 5 | Pc (No. 7) |  | 0 -1 0 | 0 0 -2 | 1 0 0 | ( $\left.\begin{array}{r}0 \\ 0 \\ -1 / 4\end{array}\right)$ | $8=2 \times 4$ | Conjugacy Class | Get irreps |
| 6 | Pc (No. 7) |  | $\left(\begin{array}{l}1 \\ 0 \\ 0\end{array}\right.$ | 1 | 0 0 2 | $\left.\begin{array}{r}0 \\ 1 / 4 \\ 0\end{array}\right)$ | $8=2 \times 4$ | Conjugacy Class | Get irreps |
| 7 | Pm (No. 6) |  | $\left(\begin{array}{l}1 \\ 0 \\ 0\end{array}\right.$ | 1 | 0 0 2 | $\left.\begin{array}{r}0 \\ 1 / 4 \\ 0\end{array}\right)$ | $8=2 \times 4$ | Conjugacy Class | Get irreps |
| 8 | $P 2_{1}$ (No. 4) |  | $\left(\begin{array}{l}1 \\ 0 \\ 0\end{array}\right.$ | 0 1 0 | 0 0 2 | ( $\left.\begin{array}{r}0 \\ 0 \\ 1 / 2\end{array}\right)$ | $8=2 \times 4$ | Conjugacy Class | Get irreps |
| 9 | $P 2_{1}$ (No. 4) |  | 0 -1 0 | 0 | 0 0 2 | $\left.\begin{array}{r}0 \\ 1 / 4 \\ -1 / 4\end{array}\right)$ | $8=2 \times 4$ | Conjugacy Class | Get irreps |
| 10 | $P \overline{1}$ (No. 2) |  | $\left(\begin{array}{l}1 \\ 0 \\ 0\end{array}\right.$ | 0 | 0 0 2 | ( $\left.\begin{array}{r}0 \\ 0 \\ 1 / 2\end{array}\right)$ | $8=2 \times 4$ | Conjugacy Class | Get irreps |
| 11 | P1 (No. 1) |  | $\left(\begin{array}{l}1 \\ 0 \\ 0\end{array}\right.$ | 0 1 0 | 0 0 2 | $\left.\begin{array}{l}0 \\ 0 \\ 0\end{array}\right)$ | $16=2 \times 8$ | Conjugacy Class | Get irreps |


| k-vectors | irreps and order parameters | isotropy subgroup transformation matrix | link to the irreps |
| :---: | :---: | :---: | :---: |
| GM: $(0,0,0)$ | $\mathrm{GM}_{1}{ }^{+}$: a ) | $\begin{gathered} \hline \text { Pnma (No. 62) } \\ \text { a,b,c;0,0,0 } \end{gathered}$ |  |
|  | $\mathrm{GM}_{3}{ }^{-}$(a) | $\mathrm{Pmc2}_{1}$ (No. 26) b,-c,-a;0,1/4,1/4 | matrices of the irreps |
| $Z:(0,0,1 / 2)$ | $Z_{2}:(a, a)$ | $\begin{gathered} \hline P c a 2_{1}(\text { No. 29) } \\ -2 c, b, a ; 0,1 / 4,-1 / 4 \end{gathered}$ | matrices of the irreps |

## Example 2: Parent space group Pnma (e)

List of subgroups that can be the result of a Landau-type transition

Go back to the input page
$\square$ Optional: Show only subgroups that can be the result of a Landau-type transition (single irrep order parameter)

## Possible limitations of the subgroup list.

(Check only one option on the left and the specific value on the right) (Check only one option on the left and the specific value on the right)

O Lowest space group to consider
O Lowest point group to consider

- Lowest crystal system to consider

1
1

$\cdots \quad \vee$
Only maximal subgroups

Further limitations considering physical properties of the point groups

- Only centrosymmetric/non centrosymmetric groups
- Only polar/non polar groups
- Only proper ferroelastic phase transitions


[^1]


## Example 2: Parent space group Pnma (f)



## Subgroups

The program Subgroups provides the possible subgroups of a space group which are possible for a given supercell. The program provides a list of the set of space groups or a graph showing the group-subgroup hierarchy, grouped into conjugacy classes. More optional information about the classes or subgroups is also given.

Enter the serial number of the space group:

Introduce the wave vector(s)
(Give the components of the wave vectors in a fractional form, $n / m$ )
$\square$
Show the independent vectors of the star
Choose the whole star of the propagation vector
More wave-vectors needed

## Example 2: Parent space group Pnma (g)

List of subgroups that fulfill the given conditions



## Example 2: Parent space group Pnma (h)

Go back to the input page

## Subgroups: Subgroups compatible with a given supercell or some propagation vector(s).

## Subgroups

The program Subgroups provides the possible subgroups of a space group which are possible for a given supercell. The program provides a list of the set of space groups or a graph showing the group-subgroup hierarchy, grouped into conjugacy classes. More optional information about the classes or subgroups is also given.
Other alternatives for the input of the program:

- Instead of the whole set of subgroups, the output can be limited to subgroups having a chosen common subgroup of lowest symmetry, common point group of lowest symmetry, or groups which belong to a specific crystal class. - Instead of a supercell, a set of modulation wave vectors can be given, including complete or partial wave-vectors stars.
- The subgroups compatible with intermediate unit cells between the unit cell of the parent space group and the given supercell (or the supercell determined by the given wave vector(s) when the previous option is used) can be included.
- When a set of wave-vectors is used as input, the output can be further refined introducing the Wyckoff positions of the atoms and/or a set of irreducible representations.



## Introduce the wave vector(s)

(Give the components of the wave vectors in a fractional form, $\mathrm{n} / \mathrm{m}$ )
$k_{1 x} 0$ $\square$ $k_{1 y} 0$ $k_{1 z} 1 / 2$

Show the independent vectors of the star
$\square$ Choose the whole star of the propagation vector
More wave-vectors needed
$\square$ Include the subgroups compatible with intermediate cells.
(It is not applied when only the maximal subgroups are calculated)

Optional: refine further the subgroups of the output giving the Wyckoff positions of the atoms
Give the Wyckoff positions
Wyckoff
$\square$ Optional: Show only subgroups that can be the result of a Landau-type transition (single irrep order parameter).
Optional: refine further the subgroups of the output giving a set of irreps
Choose the irreps

## Example 2: Parent space group Pnma (h)

The possible irreps that can describe such type of distortion

Space group: (No. 62)
Choose the irreducible representation(s) for each modulation vector

If no Wyckoff position has been given, a general position will be assumed

Possible irreducible representations

Wave-vectors of the star (1 vector):
Z:(0,0,1/2)
Descomposition of the mechanical representation(s) into irreps.
$8 \mathrm{~d}:(\mathrm{x}, \mathrm{y}, \mathrm{z}) \quad \rightarrow 6 \times \mathrm{Z} 1(2) \oplus 6 \times \mathrm{Z2}(2)$
Choose the representation(s)
irreps: $\square Z 1(2) \square Z 2(2)$
(In parentheses, the dimensions of the irreducible representations of the little group of k)

## Example 2: Parent space group Pnma (i)

Space group: (No. 62)
Choose the irreducible representation(s) for each modulation vector

If no Wyckoff position has been given, a general position will be assumed

## Possible irreducible representations

Wave-vectors of the star (1 vector):
Z:(0,0,1/2)
Descomposition of the mechanical representation(s) into irreps.
$8 \mathrm{~d}:(\mathrm{x}, \mathrm{y}, \mathrm{z}) \quad \rightarrow \quad 6 \times \mathrm{Z1}(2) \oplus 6 \times \mathrm{Z2}(2)$
Choose the representation(s)
irreps: $\quad \vee \mathrm{Z} 1(2) \square \mathrm{Z2}(2)$
(In parentheses, the dimensions of the irreducible representations of the little group of k )

## Example 2: Parent space group Pnma (i)

List of subgroups compatible which have as primary irreps all the irreps given
Get the subgroup-graph
More options


The list of possible symmetries is now reduced to three


## Example 2: Parent space group Pnma (i)

Space group: (No. 62)
Choose the irreducible representation(s) for each modulation vector

| N | Group Symbol | Transformation matrix |  |  |  | $\begin{gathered} \text { Group-Subgroup } \\ \text { index } \end{gathered}$ | Other members of the Conjugacy Class | irreps |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | Pca2 ${ }_{1}$ (No. 29) | $\left(\begin{array}{r}0 \\ 0 \\ -2\end{array}\right.$ | 0 1 0 | ${ }_{0}^{1}$ | ( $\left.\begin{array}{r}0 \\ 1 / 4 \\ -1 / 4\end{array}\right)$ | $4=2 \times 2$ | Conjugacy Class | Get irreps |
| 2 | P2 $1_{1}$ ( (No. 14) | $\left(\begin{array}{l}1 \\ 0 \\ 0\end{array}\right.$ | 0 1 0 | 0 2 | $\left.\begin{array}{l}0 \\ 0 \\ 0\end{array}\right)$ | $4=2 \times 2$ | Conjugacy Class | Get irreps |
| 3 | Pc (No.7) | $\left(\begin{array}{l}1 \\ 0 \\ 0\end{array}\right.$ |  | 2 | $\left.\begin{array}{r}0 \\ 1 / 4 \\ 0\end{array}\right)$ | $8=2 \times 4$ | Conjugacy Class | Get irreps |

If no Wyckoff position has been given, a general position will be assumed

## Possible irreducible representations

Wave-vectors of the star (1 vector):
Z:(0,0,1/2)
Descomposition of the mechanical representation(s) into irreps.
$8 \mathrm{~d}:(\mathrm{x}, \mathrm{y}, \mathrm{z}) \quad \rightarrow \quad 6 \times \mathrm{Z} 1(2) \oplus 6 \times \mathrm{Z2}(2)$
Choose the representation(s)

## irreps:

-Z1(2) $\mathrm{Z} 2(2)$
(In parentheses, the dimensions of the irreducible representations of the little group of k )


## Example 2: Parent space group Pnma (j)

Space group: (No. 62)
Choose the irreducible representation(s) for each modulation vector

If no Wyckoff position has been given, a general position will be assumed

Possible irreducible representations
Wave-vectors of the star (1 vector):
Z:(0,0,1/2)
Descomposition of the mechanical representation(s) into irreps.
$8 \mathrm{~d}:(\mathrm{x}, \mathrm{y}, \mathrm{z}) \quad \rightarrow \quad 6 \times \mathrm{Z1}(2) \oplus 6 \times \mathrm{Z2}(2)$
Choose the representation(s)
irreps: $\quad \nabla \mathrm{Z} 1(2) \vee \mathrm{Z} 2(2)$
(In parentheses, the dimensions of the irreducible representations of the little group of k )

## Example 2: Parent space group Pnma (j)

| Input data |  |
| :--- | ---: |
|  |  |
| Subgroups of the space group : | Pnma (N. 62) |
| Lowest space group to consider: | $P 1(\mathrm{~N} .1)$ |
| Modulation wave-vectors | $(0,0,1 / 2)$ |
| Irreducible representations | Z1,Z2 |
| Z:(0,0,1/2) |  |

List of subgroups compatible which have as primary irreps all the irreps given

| Get the subgroup-graph |  |  |  |  |  |  | More options |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| N | Group Symbol | Transformation matrix |  |  |  |  | Group-Subgroup index | Other members of the Conjugacy Class | irreps |
| 1 | Pc (No. 7) |  | 0 -1 0 | 0 0 -2 | 1 0 0 | ( $\left.\begin{array}{r}0 \\ 0 \\ -1 / 4\end{array}\right)$ | $8=2 \times 4$ | Conjugacy Class | Get irreps |
| 2 | $P 2_{1}$ (No. 4) |  | $\left(\begin{array}{l}1 \\ 0 \\ 0\end{array}\right.$ | 0 1 0 | 0 0 2 | ( $\left.\begin{array}{r}0 \\ 0 \\ 1 / 2\end{array}\right)$ | $8=2 \times 4$ | Conjugacy Class | Get irreps |
| 3 | $P 2_{1}$ (No. 4) |  | 0 -1 0 | 1 0 0 | 0 0 2 | $\left.\begin{array}{r}0 \\ 1 / 4 \\ -1 / 4\end{array}\right)$ | $8=2 \times 4$ | Conjugacy Class | Get irreps |
| 4 | $P \overline{1}$ (No. 2) |  | $\left(\begin{array}{l}1 \\ 0 \\ 0\end{array}\right.$ | 0 1 0 | 0 0 2 | ( $\left.\begin{array}{r}0 \\ 0 \\ 1 / 2\end{array}\right)$ | $8=2 \times 4$ | Conjugacy Class | Get irreps |
| 5 | P1 (No. 1) |  | $\left(\begin{array}{l}1 \\ 0 \\ 0\end{array}\right.$ | 0 1 0 | 0 0 2 | $\left.\begin{array}{l}0 \\ 0 \\ 0\end{array}\right)$ | 16=2x8 | Conjugacy Class | Get irreps |

The list is reduced to 5 subgroups that were not listed when choosing separately Z1 and Z 2 as single active irreps

## Example 2: Parent space group Pnma (j)

| $○$ | All irreps should be active to reach the subgroup. |
| :--- | :--- |
| $\bigcirc$ | At least one irrep should be active to reach the subgroup. |

Submit
List of subgroups compatible which have as primary irreps some of the given irreps


## Example 3: Perovskite

## $\mathrm{SrTiO}_{3}$

I4/mcm (No. 140)


$$
\left(\boldsymbol{a}_{P}+\boldsymbol{b}_{P},-\boldsymbol{a}_{P}+\boldsymbol{b}_{P}, 2 \boldsymbol{c}_{P} ; 0,0,0\right)
$$

All possible symmetries that can occur in a perovskite duet to unstable rigid-unit modes

## Example 3: Perovskite(a)

## Subgroups: Subgroups compatible with a given supercell or some propagation vector(s).

## Subgroups

The program Subgroups provides the possible subgroups of a space group which are possible for a given supercell. The program provides a list of the set of space groups or a graph showing the groupsubgroup hierarchy, grouped into conjugacy classes. More optional information about the classes or subgroups is also given.

Other alternatives for the input of the program:

- Instead of the whole set of subgroups, the output can be limited to subgroups having a chosen common subgroup of lowest symmetry, common point group of lowest symmetry, or groups which belong to a specific crystal class.


## Enter the serial number of the space group:



[^2]> 98 subgroups are possible

Space group $\operatorname{Pm} \overline{3} m$ (No. 221)

## Example 3: Perovskite (b)

Go back to the input page
Optional: refine further the subgroups of the output giving the Wyckoff positions of the atoms

Give the Wyckoff positions
Wyckoff


221
3.90643 .90643 .9064 90. 90. 90 . 3

| Sr | 1 | 1 a | 0 | 0 | 0 |
| :--- | :--- | :--- | :--- | :--- | :--- |
| Ti | 1 | 1 b | 0.5 | 0.5 | 0.5 |
| O | 1 | 3 c | 0.5 | 0.5 | 0 |

Check the type of Wyckoff positions of the atoms

|  | Multiplicity | Wyckoff Letter | Coordinates |
| :---: | :---: | :---: | :---: |
| $\square$ | 48 | n |  |
| $\square$ | 24 | m | $(x, x, z),(x,-z, x),(x, z,-x),(z, x,--x)$ $(-z, x, x),(-x, x,-z),(x,-x, z),(x,-x,-z)$ $(-x, x,-z),(-x,-x, z),(x, x,-z),(-x,-x,-z)$ $(-x, z, x),(-x,-z,-x),(z,-x, x),(-z,-x,-x)$ $(z, x, x),(x, z, x),(-x, z,-x),(-z,-x, x)$ $(-x,-z, x),(z,-x,-x),(x,-z,-x),(-z, x,-x)$ |
| $\square$ | 24 | 1 | $(1 / 2, y, z),(1 / 2,-z, y),(1 / 2, z,-y),(z, y, 1 / 2)$ $(-z, y, 1 / 2),(-y, 1 / 2, z),(y, 1 / 2, z),(1 / 2,-y,-z)$ $(1 / 2, y,-z),(1 / 2,-y, z),(y, 1 / 2,-z),(-y, 1 / 2,-z)$ $(1 / 2, z, y),(1 / 2,-z,-y),(z,-y, 1 / 2),(-z,-y, 1 / 2)$ $(z, 1 / 2, y),(y, z, 1 / 2),(-y, z, 1 / 2),(-z, 1 / 2, y)$ $(-y,-z, 1 / 2),(z, 1 / 2,-y),(y,-z, 1 / 2),(-z, 1 / 2,-y)$ |
| $\square$ | 24 | k | $\begin{gathered} (0, y, z),(0,-z, y),(0, z,-y),(z, y, 0) \\ (-z, y, 0),(-y, 0, z),(y, 0, z),(0,-y,-z) \\ (0, y,-z),(0,-y, z),(y, 0,-z),(-y, 0,-z) \\ (0, z, y),(0,-z,-y),(z,-y, 0),(-z,-y, 0) \\ (z, 0, y),(y, z, 0),(-y, z, 0),(-z, 0, y) \\ (-y,-z, 0),(z, 0,-y),(y,-z, 0),(-z, 0,-y) \end{gathered}$ |
| $\square$ | 12 | j | $(1 / 2, y, y),(1 / 2,-y, y),(1 / 2, y,-y),(y, y, 1 / 2)$ $(-y, y, 1 / 2),(-y, 1 / 2, y),(y, 1 / 2, y),(1 / 2,-y,-y)$ $(y, 1 / 2,-y),(-y, 1 / 2,-y),(y,-y, 1 / 2),(-y,-y, 1 / 2)$ |
| $\square$ | 12 | i | $\begin{gathered} (0, y, y),(0,-y, y),(0, y,-y),(y, y, 0) \\ (-y, y, 0),(-y, 0, y),(y, 0, y),(0,-y,-y) \\ (y, 0,-y),(-y, 0,-y),(y,-y, 0),(-y,-y, 0) \end{gathered}$ |
| $\square$ | 12 | h | $\begin{gathered} (x, 1 / 2,0),(x, 0,1 / 2),(0,1 / 2,-x),(0,1 / 2, x) \\ (1 / 2, x, 0),(1 / 2,-x, 0),(-x, 1 / 2,0),(-x, 0,1 / 2) \\ (0, x, 1 / 2),(1 / 2,0, x),(1 / 2,0,-x),(0,-x, 1 / 2) \\ \hline \hline \end{gathered}$ |
| $\square$ | 8 | g | $\begin{gathered} (x, x, x),(x,-x, x),(x, x,-x),(-x, x, x) \\ (x,--x,-x),(-x, x,-x),(-x,-x, x),(-x,-x,-x) \end{gathered}$ |
| $\square$ | 6 | f | $\begin{aligned} & (x, 1 / 2,1 / 2),(1 / 2,1 / 2,-x),(1 / 2,1 / 2, x) \\ & (1 / 2, x, 1 / 2),(1 / 2,-x, 1 / 2),(-x, 1 / 2,1 / 2) \end{aligned}$ |
| $\square$ | 6 | e | $\begin{aligned} & (x, 0,0),(0,0,-x),(0,0, x) \\ & (0, x, 0),(0,-x, 0),(-x, 0,0) \end{aligned}$ |
| - | 3 | d | $(1 / 2,0,0),(0,0,1 / 2),(0,1 / 2,0)$ |
| $\square$ | 3 | c | (0,1/2,1/2),(1/2,1/2,0),(1/2,0,1/2) |
| $\square$ | 1 | b | (1/2,1/2,1/2) |
| $\square$ | 1 | a | $(0,0,0)$ |

$\square$ Do not consider subgroups attainable only through strain-like distortions

## Example 3: Perovskite (c)

| Enter the serial number of the space group: |
| :--- |

List of possible subgroups assuming the given wyckoff positions


[^3]$\square$ Optional: Show only subgroups that can be the result of a Landau-type transition (single irrep order parameter).

## Wyckoff positions of the atoms

3d:(1/2,0,0)
1b:(1/2,1/2,1/2)
1a: $(0,0,0)$

Possible limitations of the subgroup list.
(Check only one option on the left and the specific value on the right)
(Check only one option on the left and the specific value on the right)

- Lowest space group to consider
- Lowest crystal system to consider
- Only maximal subgroups



## Example 3: Perovskite (d)

Subgroups that belong to the same conjugacy class, limited to those compatible with the given supercell or the supercell determined by the given wave vector(s).

| N | Group Symbol | Transformation matrix | Group-Subgroup index | Symmetry operations | Set of subgroups* | irreps |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 5.1 | 14/mcm (No. 140) | $\left(\begin{array}{rrrr}1 & -1 & 0 & 0 \\ 0 & 0 & -2 & 0 \\ 1 & 1 & 0 & 0\end{array}\right)$ | $6=2 \times 3$ | Plain text format <br> Matrix form | List of subgroups <br> Graph of subgroups | Get irreps |
| 5.2 | 14/mcm (No. 140) | $\left(\begin{array}{rrrr}0 & 0 & 2 & 0 \\ 1 & 1 & 0 & 0 \\ -1 & 1 & 0 & 0\end{array}\right)$ | $6=2 \times 3$ | Plain text format <br> Matrix form | List of subgroups <br> Graph of subgroups | Get irreps |
| 5.3 | 14/mcm (No. 140) | $\left(\begin{array}{rrrr}1 & 1 & 0 & 0 \\ -1 & 1 & 0 & 0 \\ 0 & 0 & 2 & 0\end{array}\right)$ | $6=2 \times 3$ | Plain text format <br> Matrix form | List of subgroups <br> Graph of subgroups | Get irreps |

* List or graph of subgroups that are related with the chosen group through group-subgroup relation.


## Example 3: Perovskite (e)

| Group $\rightarrow$ subgroup | Transformation matrix |
| :---: | :---: |
| $P m \overline{3} m($ N. 221 $) \rightarrow I 4 / m c m ~(N .140)$ | $\left(\begin{array}{rrrr} \\ & \left(\begin{array}{rlll}1 & 1 & 0 & 0 \\ -1 & 1 & 0 & 0 \\ 0 & 0 & 2 & 0\end{array}\right) \\ \hline\end{array}\right.$ |

Representations and order parameters

Show the graph of isotropy subgroups

| k-vectors | irreps and order parameters | isotropy subgroup transformation matrix | link to the irreps |
| :---: | :---: | :---: | :---: |
| GM: (0,0,0) | $\mathrm{GM}_{1}{ }^{+}$: ${ }^{\text {a }}$ ) | $\begin{gathered} \hline \hline P m \overline{3} m(\text { No. 221) } \\ \text { a,b,c;0,0,0 } \end{gathered}$ |  |
|  | $\mathrm{GM}_{3}{ }^{\text {: }}(\mathrm{a}, 0)$ | $\begin{gathered} \hline \hline \text { P4/mmm (No. 123) } \\ \text { a,b,c;0,0,0 } \end{gathered}$ | matrices of the irreps |
| R: (1/2,1/2,1/2) | $\mathrm{R}_{4}{ }^{+}:(\mathrm{a}, 0,0)$ | 14/mcm (No. 140) a-b,a+b,2c;0,0,0 | matrices of the irreps |

## Example 3: Perovskite (f)



Representations and order parameters

Show the graph of isotropy subgroups

| k-vectors | irreps and order parameters | isotropy subgroup transformation matrix | link to the irreps |
| :---: | :---: | :---: | :---: |
| GM: $(0,0,0)$ | $\mathrm{GM}_{1}{ }^{+}$: (a) | $\begin{gathered} P m \overline{3} m(\text { No. 221) } \\ \text { a,b,c;0,0,0 } \end{gathered}$ |  |
|  | $\mathrm{GM}_{3}{ }^{\text {: }}$ (a,-( $\left.\left.\sqrt{3} \mathrm{a}\right)\right)$ | P4/mmm (No. 123) a,-c,b;0,0,0 | matrices of the irreps |
| R: (1/2,1/2,1/2) | $\mathrm{R}_{4}{ }^{+}:(0,0, a)$ | $\begin{aligned} & 14 / \mathrm{mcm}(\text { No. 140) } \\ & \mathrm{a}+\mathrm{c},-\mathrm{a}+\mathrm{c},-2 \mathrm{~b} ; 0,0,0 \end{aligned}$ | matrices of the irreps |


| Group $\rightarrow$ subgroup | Transformation matrix |  |  |
| :---: | :---: | :---: | :---: |
| $\operatorname{Pm} \overline{3} m($ N. 221 $) \rightarrow / 4 / m c m ~(N .140)$ | $\left(\begin{array}{rrrr}0 & 0 & 2 & 0 \\ 1 & 1 & 0 & 0 \\ -1 & 1 & 0 & 0\end{array}\right)$ |  |  |

Representations and order parameters

| k-vectors | irreps and order parameters | isotropy subgroup transformation matrix | link to the irreps |
| :---: | :---: | :---: | :---: |
| GM: $(0,0,0)$ | $\mathrm{GM}_{1}^{+}$: (a) | $\begin{gathered} \hline P m \overline{3} m(\text { No. 221) } \\ \text { a,b, } ; ; 0,0,0 \end{gathered}$ |  |
|  | $\mathrm{GM}^{+}:(\mathrm{a}, \sqrt{3} \mathrm{a})$ | $\begin{gathered} \hline \text { P4/mmm (No. 123) } \\ -\mathrm{c}, \mathrm{~b}, \mathrm{a} ; 0,0,0 \end{gathered}$ | matrices of the irreps |
| R: (1/2,1/2,1/2) | $\mathrm{R}_{4}{ }^{+}:(0, a, 0)$ | $\begin{aligned} & \hline 14 / \mathrm{mcm}(\text { No. } 140) \\ & \mathrm{b}-\mathrm{c}, \mathrm{~b}+\mathrm{c}, 2 \mathrm{a} ; 0,0,0 \end{aligned}$ | matrices of the irreps |

The direction changed for the $R_{4}^{+}$

## Example 3: Perovskite (g)



| - | b | t | $(1 / 2, x, 1 / 2),(1 / 2,-x, 1 / 2),(-\mathrm{x}, 1 / 2,1 / 2)$ |
| :---: | :---: | :---: | :---: |
| $\square$ | 6 | e | $(\mathrm{x}, 0,0),(0,0,-\mathrm{x}),(0,0, \mathrm{x})$ <br> $(0, x, 0),(0,-x, 0),(-\mathrm{x}, 0,0)$ |
| $\square$ | 3 | d | $(1 / 2,0,0),(0,0,1 / 2),(0,1 / 2,0)$ |
| $\square$ | 3 | c | $(0,1 / 2,1 / 2),(1 / 2,1 / 2,0),(1 / 2,0,1 / 2)$ |
| $\square$ | 1 | b | $(1 / 2,1 / 2,1 / 2)$ |
| $\square$ | 1 | a | $(0,0,0)$ |

$\square$ Do not consider subgroups attainable only through strain-like distortions

[^4]Subgroups: Subgroups compatible with a given supercell or some propagation vector(s).

## Subgroups

The program Subgroups provides the possible subgroups of a space group which are possible for a given supercell. The program provides a list of the set of space groups or a graph showing the group-subgroup hierarchy, grouped into conjugacy classes. More optional information about the classes or subgroups is also given

Other alternatives for the input of the program:

- Instead of the whole set of subgroups, the output can be limited to subgroups having a chosen common subgroup of lowest symmetry, common point group of lowest symmetry, or groups which belong to a specific crystal class.
- Instead of a supercell, a set of modulation wave vectors can be given, including complete or partial wavevectors stars.
- The subgroups compatible with intermediate unit cells between the unit cell of the parent space group and the given supercell (or the supercell determined by the given wave vector(s) when the previous option is used) can be

Enter the serial number of the space group:

| choose it 221 |
| :---: |

Introduce the wave vector(s)
(Give the components of the wave vectors in a fractional form, $\mathrm{n} / \mathrm{m}$ )
$k_{1 \times} \quad 1 / 2$ $\square$ $\mathrm{k}_{1 \mathrm{y}} 1 / 2$ $\qquad$ $\mathrm{k}_{1 \mathrm{z}} 1 / 2$
Show the independent vectors of the star
$\square$ Choose the whole star of the propagation vector
More wave-vectors needed

[^5]Optional: refine further the subgroups of the output giving the Wyckoff positions of the atoms Give the Wyckoff positions

## Example 3: Perovskite (g)

## Go back to the input page

Subgroups: Subgroups compatible with a given supercell or some propagation vector(s).

## Subgroups

The program Subgroups provides the possible subgroups of a space group which are possible for a given supercell. The program provides a list of the set of space groups or a graph showing the group-subgroup hierarchy, grouped into conjugacy classes. More optiona information about the classes or subgroups is also given.

Other alternatives for the input of the program:

- Instead of the whole set of subgroups, the output can be limited to subgroups having a chosen common subgroup of lowest symmetry, common point group of lowest symmetry, or groups which belong to a specific crystal class.
- Instead of a supercell, a set of modulation wave vectors can be given, including complete or partial wavevectors stars.
- The subgroups compatible with intermediate unit cells between the unit cell of the parent space group and the given supercell (or the supercell determined by the given wave vector(s) when the previous option is used) can be included.
- When a set of wave-vectors is used as input, the output can be further refined introducing the Wyckoff positions of the atoms and/or a set of irreducible representations.

Tutorial_SUBGROUPS: download

See the Help for details
Space group: $\quad P m \overline{3} m$ (No. 221)

| $k_{1}=(1 / 2,1 / 2,1 / 2)$ |
| :--- |

$\square$ Include the subgroups compatible with intermediate cells.
(It is not applied when only the maximal subgroups are calculated)

```
Wyckoff positions of the atoms
3d:(1/2,0,0)
1b:(1/2,1/2,1/2)
1a:(0,0,0)
```

$\square$ Optional: Show only subgroups that can be the result of a Landau-type transition (single irrep order parameter)

Optional: refine further the subgroups of the output giving a set of irreps
Choose the irreps
Representations

Optional: possible limitations of the subgroup list
(Check only one option on the left and the specific value on the right)
(Check only one option on the left and the specific value on the right)

- Lowest space group to consider
- Lowest point group to consider

Lowest crystal system to consider
choose it


Only maximal subgroups

## Example 3: Perovskite (g)

> Space group: $\operatorname{Pm} \overline{3} m$ (No. 221)
> Choose the irreducible representation(s) for each modulation vector

> If no Wyckoff position has been given, a general position will be assumed

> Non bolded irreps are incompatible with the given Wyckoff positions Bolded irreps are compatible with at least one given Wyckoff position Red colored irreps are compatible with all the Wyckoff positions given

> Possible irreducible representations

Wave-vectors of the star (1 vector):
R:(1/2,1/2,1/2)
Descomposition of the mechanical representation(s) into irreps.

```
3d:(1/2,0,0) }\quad->\quad1\timesR1+(1)\oplus1\timesR3+(2) \oplus1\timesR4+(3)\oplus1\timesR5+(3
1b:(1/2,1/2,1/2) }\quad->\quad1\timesR5+(3
```



Choose the representation(s)
irreps: $\quad \square \mathbf{R 1 + ( 1 )} \quad$ R1-(1) $\quad$ R2+(1) $\quad$ R2-(1) $\square \mathbf{R 3 + ( 2 )} \quad$ R3-(2) $\checkmark \mathbf{R 4 + ( 3 )} \square \mathbf{R 4 - ( 3 ) ~} \square \mathbf{R 5 + ( 3 )} \quad$ R5-(3)
(In parentheses, the dimensions of the irreducible representations of the little group of $k$ )

Only 4 of the 10 possible irreps are relevant

## Example 3: Perovskite (g)

Subgroups: Subgroups compatible with a given supercell or some propagation vector(s).

## Subgroups

The program Subgroups provides the possible subgroups of a space group which are possible for a given supercell. The program provides a list of the set of space groups or a graph showing the group-subgroup hierarchy, grouped into conjugacy classes. More optiona information about the classes or subgroups is also given
Other alternatives for the input of the program

- Instead of the whole set of subgroups, the output can be limited to subgroups having a chosen common subgroup of lowest symmetry, common point group of lowest symmetry or groups which belong to a specific crystal class
- Instead of a supercell, a set of modulation wave vectors can be given, including complete or partial wave vectors stars.
- The subgroups compatible with intermediate unit cells between the unit cell of the parent space group and the given supercell (or the supercell determined by the given wave vector(s) when the previous option is used) can b included.
- When a set of wave-vectors is used as input, the output can be further refined introducing the Wyckoff positions of the atoms and/or a set of irreducible representations.

Tutorial_SUBGROUPS: download

See the Help for details

```
Space group:
Pm\overline{3}m(No. 221)
```

$k_{1}=(1 / 2,1 / 2,1 / 2) \quad$ Set of chosen modulation wave-vectors
$\square$ Include the subgroups compatible with intermediate cells.
(It is not applied when only the maximal subgroups are calculated)

```
Wyckoff positions of the atoms
3d:(1/2,0,0)
1b:(1/2,1/2,1/2)
1a:(0,0,0)
```


## Chosen representations

R: $(1 / 2,1 / 2,1 / 2)$
R4+(3)
Optional: possible limitations of the subgroup list
(Check only one option on the left and the specific value on the right)
(Check only one option on the left and the specific value on the right)
Lowest space group to consider
Lowest point group to consider
Lowest crystal system to consider
Only maximal subgroups

Optional: further limitations considering physical properties of the point groups

- Only centrosymmetric / non-centrosymmetric groups
- Only polar / non-polar groups


Only proper ferroelastic phase transitions
This option is incompatible with the previous two options and with the selection of representations)

## Example 3: Perovskite (g)

List of possible subgroups assuming the given wyckoff positions and that have as primary irreps all the irreps given


All possible distinct space group symmetries (subgroups of the parent $\operatorname{Pm} \overline{3} m$ ) that can result from an order parameter with R4+ symmetry.

## Example 3: Perovskite (h)

All possible symmetries that a perovskite can acquire as the result of the freezing of some combinations of the three R4+
 distortions

## Example 3: Perovskite (k)

SUBGROUPS can be combined with the program TRANSTRU to create an initial structural model of a distorted structure under one or more of the symmetries obtained with SUBGROUPS.

Create the CIF file for the $\mathrm{C} 2 / \mathrm{c}$ structure observed in $\mathrm{LaCoO}_{3}$.

| N | Group Symbol | Transformation matrix |  |  |  | $\begin{array}{\|c\|} \text { Group-Subgroup } \\ \text { index } \end{array}$ | Other members of the Conjugacy Class | irreps |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | $R \overline{3} C$ ( No .167 ) | $\left(\begin{array}{rr}-1 & \\ 0 & - \\ 1 & -\end{array}\right.$ | $\begin{array}{rrr}0 \\ -1 & -2 \\ -1 & -2\end{array}$ | 2 -2 2 | $\left.\begin{array}{l}0 \\ 0 \\ 0\end{array}\right)$ | $8=2 \times 4$ | Conjugacy Class | Get irreps |
| 2 | 14/mcm (No. 140) | $\left(\begin{array}{ll}1 & -1 \\ 0 \\ 1\end{array}\right.$ | -1  <br> 0 -2 <br> 1  | 0 -2 0 | $\left.\begin{array}{l}0 \\ 0 \\ 0\end{array}\right)$ | $6=2 \times 3$ | Conjugacy Class | Get irreps |
| 3 | Imma (No. 74) | $\left(\begin{array}{l}0 \\ 0 \\ 2\end{array}\right.$ | $\begin{array}{ll}1 & \\ 1 & - \\ 0 & \\ 0\end{array}$ | -1 1 0 | $\left.\begin{array}{r}1 / 2 \\ 0 \\ 1 / 2\end{array}\right)$ | $12=2 \times 6$ | Conjugacy Class | Get irreps |
| 4 | C2/c (No. 15) | $\left(\begin{array}{r}1 \\ -2 \\ -1\end{array}\right.$ | 1  <br> 0 - <br> 1  | -1 0 1 | $\left.\begin{array}{r}1 / 2 \\ 1 / 2 \\ 0\end{array}\right)$ | $24=2 \times 12$ | Conjugacy Class | Get irreps |
| 5 | C2/m (No. 12) | $\left(\begin{array}{l}2 \\ 0 \\ 0\end{array}\right.$ | 0 0 -2 | -1 1 0 | $\left.\begin{array}{r}0 \\ 1 / 2 \\ 1 / 2\end{array}\right)$ | $24=2 \times 12$ | Conjugacy Class | Get irreps |
| 6 | $P \overline{1}$ (No. 2) | $\left(\begin{array}{rr}1 \\ 0 & -1 \\ -1\end{array}\right.$ | -1 1 1 0 | 0 1 1 | $\left.\begin{array}{l}0 \\ 0 \\ 0\end{array}\right)$ | $48=2 \times 24$ | Conjugacy Class | Get irreps |



## Example 3: Perovskite (k)

## https://www.cryst.ehu.es/cryst/transtru.html

## Transform Structure

## Transform Structure

TRANSTRU can transform a structure in two ways:

- To a lower symmetry space group. The transformed structure is given in the low symmetry space group basis, taking care of all possible splittings of the Wyckoff positions.
- With an arbitrary matrix. The structure, including the cell parameters and the atoms in the unit cell, is transformed with an arbitrary matrix introduced by the user.

Only the default choice for the conventional setting of the space groups is used.

| Structure Data [in CIF format] High Symmetry Structure | Browse... No file selected. <br> HINT: [ The option for a given filename is preferential ] |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $\begin{array}{\|lllll\|} \hline 221 & & & \\ 3.9064 & 3.9064 & 3.9064 & 90 . & \\ 3 & & & \\ 3 & & & \\ \text { La } & 1 & 1 \mathrm{a} & 0.000000 \\ \text { Co } & 1 & 1 \mathrm{~b} & 0.500000 \\ 0 & 1 & 3 \mathrm{c} & 0.500000 \end{array}$ |  |  |  | 0.000000 <br> 0.500000 <br> 0.500000 | 0.000000 <br> 0.500000 <br> 0.000000 |
|  |  |  |  |  |  |  |
|  | Transform structure to a subgroup basis <br> Transform structure with an arbitrary matrix |  |  |  |  |  |
|  |  |  |  |  |  |  |

## Example 3: Perovskite (k)

Transform Structure

## Transform Structure

TRANSTRU transforms the structure to the low symmetry space group basis, taking care of all possible splittings of the Wyckoff positions.


## Example 3: Perovskite (k)

## Transform structure

Transformation matrix: $a+2 b-c,-a-c+1 / 2,-a+c+1 / 2$
High symmetry structure

| 221 |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 3.90643 .90643 .9064 90. 90. 90. |  |  |  |  |  |
| 3 |  |  |  |  |  |
| La | 1 a |  | 000000 | 0.000000 | 0.000000 |
| Co | 1b |  | 500000 | 0.500000 | 0.500000 |
| $\bigcirc$ | 3 c |  | 500000 | 0.500000 | 0.000000 |
| Visualize this structure |  | CIF File | Cartesian Coordinates |  |  |
| Low symmetry structure |  |  |  |  |  |
| 015 |  |  |  |  |  |
| 9.568686 | 65.524484 | 5.524484 | 90.00000 | 125.264397 | 90.000000 |
|  |  |  |  |  |  |
| La | 14 c |  | 750000 | 0.250000 | 0.500000 |
| Co | 14 e |  | 000000 | 0.750000 | 0.750000 |
| $\bigcirc$ | 1 4a |  | 000000 | 0.000000 | 0.500000 |
| $\bigcirc$ | $1 \_24 \mathrm{~b}$ |  | 000000 | 0.500000 | 0.000000 |
| - | $1{ }^{-3} 4 \mathrm{~d}$ |  | 250000 | 0.250000 | 0.500000 |

Visualize this structure CIF File

Cartesian Coordinates

You can download directly the CIF file with the atomic positions corresponding to the ideal perovskite structure


[^0]:    O List of subgroups

[^1]:    O List of subgroups
    Oraph of subgroups

[^2]:    - List of subgroups
    - Graph of subgroups

[^3]:    $\square$ Include the subgroups compatible with intermediate cells. (It is not applied when only the maximal subgroups are calculated)

[^4]:    Submit

[^5]:    $\square$ Include the subgroups compatible with intermediate cells
    (It is not applied when only the maximal subgroups are calculated)

